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Observer-based fuzzy integral sliding mode control for bilateral teleoperation systems with time-varying delays

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ABSTRACT

This paper aims to examine the stability and tracking performance of a bilateral teleoperation system. The Takagi–Sugeno fuzzy method is utilized, through which the nonlinear master–slave dynamics is converted as a fuzzy-based system. The state observers are designed for the linearized fuzzy teleoperation systems, and the corresponding estimation errors are formulated. Importantly, a novel observer-based fuzzy integral sliding mode control is developed by deliberately introducing the delay term into the sliding surfaces. As such, advanced delay-product type of Lyapunov–Krasovskii functionals are constructed for the augmented state vectors, in order to acquire the additional delay information. In addition, the Wirtinger-based integral inequality along with an extended reciprocally convex matrix inequality is applied to the Lyapunov derivatives to establish the delay-dependent stability conditions. Numerical results are provided to demonstrate efficacy of the developed control mechanism.

1. Introduction

Due to the rapid advancement of science and technology, robots are becoming a significant element of manufacturing and lifestyle in our daily activities [11,39]. A bilateral teleoperator is a dual-robot system that utilizes communication channels to transmit sensed and command signals among the master and slave robot. In controlling a bilateral teleoperation system comprising both master–slave robotic manipulators, one of the key objectives is that tracking and synchronization with respect to position, velocity, and force responses have to be achieved effectively, in order to complete the desired task. Since the manipulators are subject to a high risk of environmental disturbances and modeling uncertainties, their tracking and synchronization performances can be compromised to a large extent. Furthermore, due to certain limitations like packet dropouts, network congestion, limited bandwidth, and noise, it is inevitable that time delays in signal transmission occur between the master and slave devices through the communication links [1,5,9,34]. However, because of the intricate nature of communication networks, the delays of data packets are not only different for the forward and backward channels but also time-varying. This is because a teleoperation system fundamentally uses dynamic and time-varying network bandwidths, and data transmission in the network experiences time-varying delays. Furthermore, the forward and backward delays may not be equivalent because the data packets in the forward and backward channels can take different network pathways; therefore, affecting stability of the teleoperation system [29,38,40]. These issues must be addressed appropriately to realize the implementation of a stable teleoperation system with enhanced outcomes.

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In recent years, modeling using the Takagi–Sugeno (T–S) fuzzy approach, especially in tackling higher nonlinear systems, is found to be simple and very effective [4,15,32]. For teleoperation systems involving nonlinear terms, the T–S fuzzy approach provides an approximate degree of accuracy in linearizing the system as a set of linear sub-models. The T–S method can be blended with other control strategies easily for developing suitable controllers to handle complex nonlinear systems. Furthermore, for most nonlinear models, the system state information are often unavailable in practice. To overcome this problem, an estimation on the state variable is imposed through an observer design [14,20,25,27]. Observer-based control have been applied to nonlinear switched systems using finite-time control in [7] and adaptive output feedback control in [8]. Therefore, designing and developing an observer-based control for analyzing complex systems with high nonlinear dynamics is significant from both theoretical and practical aspect.

The sliding mode control (SMC) technique is effective for solving numerous real-world problems, in view of its advantages of being insensitive to model uncertainties and perturbations to deliver strong robustness resulting in improved performance. In SMC, the designed sliding surface provides an ideal sliding motion behavior in the sense that the controller can drive the system onto the given sliding mode surface [10,24,26]. However, a system incorporated with SMC still undergoes uncertainties and disturbances during the reaching phase. In the event of an external disturbance when the common SMC follows any trajectory, a steady-state error can occur, making it impossible to fulfill the desired performance index. Through switching of control variables, the integral SMC (ISMC) method causes the system state to slide over the sliding mode surface rendering the system invariant whenever it is disrupted by some external disturbances. It reduces chattering and enables quick responses while minimizing the structural complexity of the fuzzy control system. Thus, an ISMC mechanism is developed to improve robustness for the overall response. In ISMC, the system trajectories commence from the sliding surface itself, in order to eliminate the reaching phase [13,21]. Hence, depending on practical requirements, different SMC structures integrated with other control techniques such as fuzzy logic, adaptive control, neural networks etc. can be developed. Recently, the fuzzy-based ISMC technique has gained significant research attention (e.g. see [18,28,35,45]). The authors in [21] investigated the nonlinear descriptor systems through observer-based fuzzy ISMC. In this research, we are motivated to design and develop an observer-based fuzzy ISMC with time-varying delays in the sliding surface for a master–slave teleoperation system.

With respect to Lyapunov–Krasovskii functional (LKF) approach, the delay partitioning method and augmented LK methods are widely studied to undertake stability issues with time-varying delays [3,41,42,44]. The most well-known and efficient approach to analyze dynamic systems with a time-varying delay is the LKF method, which can result in stability requirements in the form of linear matrix inequalities (LMIs). The key concept is to choose a positive definite functional that ensures its derivative along the system's trajectory in regard to time is negative definite. Unquestionably, the most important factor in determining a stability criterion is the selection of an adequate LKF. Recently, delay-product-type of LKFs consisting of non-integral quadratic terms and integral terms have been constructed in [22,23]. The advantage of utilizing the delay-product-type of LKFs lies in the fact that the delay is multiplied with non-integral terms, such that the delay information is fully utilized to derive the stability criteria. Furthermore, the Wirtinger-based integral inequality coupled with the extended reciprocally convex matrix inequality (ERCMI) to the derivatives acquired from delay-product-type terms is able to provide useful additional information.

Motivated by these facts, we derive the delay-dependent stability criteria pertaining to a bilateral teleoperation system with an observer-based fuzzy ISMC via delay-product-type of LKFs in this study. The main contributions of our work are summarized as follows:

- Using the T-S fuzzy inference technique, the nonlinear master and slave teleoperation systems are linearized for which their corresponding state observer systems are proposed through a parallel distribution compensation method. Different from the existing methods, an observer based fuzzy ISMC incorporating time-varying delays in the sliding surface is formulated for the master–slave teleoperation system. The fundamental advantage of ISMC is its capability in eliminating the reaching phase required in a traditional SMC approach. This allows sliding mode motion to be applied from the very beginning of control actions while maintaining the order of the original model and providing guarantee on the system robustness during the entire response phase. As a result, the influence of time-varying delays toward the stability analysis can be effectively addressed.
- Next, to address the aforementioned delay effects, we construct a new delay-product type of LKF candidate from which more delay information can be acquired for deriving the delay-dependent stability criteria. The LKF construction in our proposed method multiplies time-varying delay terms with non-integral terms, leading to an LKF approach that possesses additional time delay information. Furthermore, because the constraint in certain aspect of the underlying system is relaxed when using the delay-product-type functional approach, the LKF can assume a broad shape. Indeed, the delay-product-type functional has many advantages that warrant more research. Moreover, the Wirtinger-based integral inequality [30] and ERCMI [43] are employed to address the delay terms in the LKF derivatives. The Wirtinger inequality enables the availability of precise integral inequality to be considered in our proposed approach.
- To achieve synchronization and better tracking performance, H_∞ design pertaining to a prescribed performance index γ is utilized for deriving the stability conditions. In this respect, the linear matrix inequalities (LMIs) in MATLAB are utilized for the stability analysis. Comprehensive simulations on master–slave manipulators with 2-DoFs incorporating environmental forces together with a non-passive human operator are conducted.

Different controllers, including ISMC, adaptive coordination control, and others, have been discussed in the literature for use in bilateral teleoperation systems [2,3,9,17,38,45]. To create a delay-dependent stability condition in our study, we concentrate on observer-based fuzzy ISMC. To the best of our knowledge, this control strategy has not been developed and described in the literature. Specifically, an observer based fuzzy ISMC method by considering time-varying delays in the sliding surface are constructed and the delay-product type of LKF candidate is considered to address time-varying delay effects in this study. Furthermore, the LMIs are obtained to analyze the system stability. Our proposed method is new, which contributed toward advancing research and application of teleoperated systems.

2. Model formulation and preliminaries

2.1. Notations

The following notations have been used in this article. $\mathbb{R}$ denotes the set of positive real numbers. $\mathbb{R}^n$ represents the n-dimensional Euclidean space over $\mathbb{R}$. $\mathbb{R}^{n\times n}$ represents the set of all $n\times n$ real matrices. In addition, $*$ denotes the symmetric block in the block diagonal matrix. Subscripts m and s represent the master and slave system, respectively. The set of all real symmetric matrices of dimensions $n\times m$ is represented as $\mathbb{S}^{n\times m}$. $\mathbb{S}_n$ and $\mathbb{S}_n^+$ denote the sets of symmetric and symmetric positive definite matrices, respectively.

2.2. Description of the system

The master–slave manipulators having nonlinear Lagrangian dynamics can be expressed as follows:

$$M_m(q_m)\ddot{q}_m + C_m(q_m,\dot{q}_m)\dot{q}_m + G_m(q_m) = \tau_m + F_h,$$
$$M_s(q_s)\ddot{q}_s + C_s(q_s,\dot{q}_s)\dot{q}_s + G_s(q_s) = \tau_s - F_s. \tag{1}$$

where $i=\{m,s\}$, $q_i,\dot{q}_i,\ddot{q}_i\in\mathbb{R}^n$ denote the vectors with respect to the joint position, velocity, and acceleration of the master–slave robots; $M_i(q_i)\in\mathbb{R}^n$ and $C_i(q_i,\dot{q}_i)\in\mathbb{R}^n$ denote the matrices of inertia, as well as the Coriolis and centrifugal effects, respectively. In addition, the control and gravitational torques are denoted by $\tau_i\in\mathbb{R}^n$ and $G_i(q_i)\in\mathbb{R}^n$, respectively. On the other hand, the environmental force exerted on the slave manipulator and the human operator force are denoted as $F_s\in\mathbb{R}^n$ and $F_h\in\mathbb{R}^n$, respectively.

Remark 1. It should be noted that in this article, the force of a human operator on the master controller is built to adapt to the feedback pressure applied by the slave environment. The human operator can therefore sense the situation in which improved transparency could be attained. Furthermore, a bilateral teleoperation instructs both master and slave robots to monitor each other's position/velocity, guaranteeing system stability and improved performance. A force sensor is employed on the slave end of a position-force control architecture to communicate the contact force from the slave's environment back to the master, generating kinesthetic force feedback to the system.

In general, we can represent a nonlinear system with a fuzzy model. Through the fuzzy membership functions, a piecewise interpolation pertaining to some linear models can be derived. By exploiting the T-S fuzzy model, these linear models can be combined together. Specifically, we can express the jth rule using the following IF-THEN construct [16]:

Plant rule j: IF $\theta_1(t)$ is N_{j1} *and* $\cdots$ *and* $\theta_p(t)$ is N_{jp}
THEN

$$\dot{x}(t) = A_j x(t) + B_j\tau(t),$$
$$y(t) = C_j x(t), \tag{2}$$

where $x(t)\in\mathbb{R}^n$, $y(t)\in\mathbb{R}^n$, and $\theta(t)=\left[\theta_1(t),\dots,\theta_p(t)\right]$ constitute the state vector, system output, and premise variables, respectively. While, the notation B_j denotes the input, C_j represents the output and A_j denotes the system matrix. It should be noted that while the system matrix, A_j, is constant, the definition of the B_j and C_j matrices vary depending on whether zeros are present or absent in the master or slave subsystems. In addition, the control torque, fuzzy set, and number of fuzzy IF-THEN rules are denoted by $\tau(t)\in\mathbb{R}^n$, N_{jk}, and $j=1,2,\dots,q$, respectively.

Let us define $x_{i1}=q_{i1}, x_{i2}=\dot{q}_{i1}, x_{i3}=q_{i2}, x_{i4}=\dot{q}_{i2}$, $i=\{m,s\}$. As such, we can use a state–space form to express (1) as:

$$\dot{x}_m = f(x_m) + g(x_m)(\tau_m(t)+F_h(t)),$$
$$\dot{x}_s = f(x_s) + g(x_s)(\tau_s(t)-F_s(t)), \tag{3}$$

where $x_i=\begin{bmatrix}x_{i1} & x_{i2} & x_{i3} & x_{i4}\end{bmatrix}, \tau_i=\begin{bmatrix}\tau_{i1} & \tau_{i2}\end{bmatrix}$,

$$f(x_i)=\begin{bmatrix}x_{i2}\\ x_{i4}\\ f_1(x_i)\\ f_2(x_i)\end{bmatrix}, g(x_i)=\begin{bmatrix}0 & 0\\ 0 & 0\\ g_{11}(x_i) & g_{12}(x_i)\\ g_{21}(x_i) & g_{22}(x_i)\end{bmatrix}.$$

Note that the values of $f_1(x_i), f_2(x_i), g_{11}(x_i), g_{12}(x_i), g_{21}(x_i), g_{22}(x_i)$ are given in the numerical simulation section.

The controllers $\tau_m(t)$ and $\tau_s(t)$ are considered with gravity compensation with the control input $u_i(t)$, as follows:

$$\tau_m(t) = u_m(t) + G_m(q_m), \qquad \tau_s(t) = u_s(t) + G_s(q_s).$$

For given pairs of $(x_m(t),\tau_m(t)), (x_s(t),\tau_s(t))$, we can infer the final defuzzified outputs of fuzzy system (3) as:

Plant rule j: If $\theta_{i1}(t)$ is N_{j1} and $\cdots$ and $\theta_{ip}(t)$ is N_{jp} then

$$\text{Master:}\begin{cases}\dot{x}_m(t) = \sum_{j=1}^{q}\lambda_j(\theta_m(t))\left[A_{mj}x_m(t)+B_{mj}(u_m(t)+F_h(t))\right]\\ y_m(t) = \sum_{j=1}^{q}\lambda_j(\theta_m(t))C_{mj}x_m(t)\end{cases} \tag{4}$$

$$\text{Slave:} \begin{cases} \dot{x}_s(t) = & \sum_{j=1}^{q} \lambda_j(\theta_s(t)) \left[A_{sj} x_s(t) + B_{sj}(u_s(t) - F_s(t)) \right] \\ y_s(t) = & \sum_{j=1}^{q} \lambda_j(\theta_s(t)) C_{sj} x_s(t) \end{cases} \tag{5}$$

For $i = \{m, s\}$, $\lambda_j(\theta_i(t)) = \frac{h_j(\theta_i(t))}{\sum_{j=1}^{q} h_j(\theta_i(t))}$, $h_j(\theta_i(t)) = \Pi_{k=1}^{p} N_{jk}(\theta_i(t))$, and $N_{jk}(\theta_i(t))$ is the grade of membership of $\theta_{ik}(t) = [\theta_{i1}, \theta_{12}, \ldots, \theta_{ip}]$ in N_{jk}. Now assume that, $h_j(\theta_i(t)) \geq 0$ for $j = 1, \ldots, q$ and $\sum_{j=1}^{q} h_j(\theta_i(t)) > 0$ for all t. Therefore, $\lambda_j(\theta_i(t)) \geq 0$ for $j = 1, \ldots, q$ and $\sum_{j=1}^{q} \lambda_j(\theta_i(t)) = 1$.

The design of a fuzzy-based state observer for the master–slave system is performed through a parallel distributed compensation method. Therefore, the overall fuzzy observer for the T-S model (4) and (5) are:

Observer rule j: If $\theta_{i1}(t)$ is N_{j1} and $\cdots$ and $\theta_{ip}(t)$ is N_{jp} then

$$\text{Master:} \begin{cases} \dot{\hat{x}}_m(t) = & \sum_{j=1}^{q} \lambda_j(\theta_m(t)) \left[A_{mj} \hat{x}_m(t) + B_{mj} u_m(t) + L_{mj}(y_m(t) - \hat{y}_m(t)) \right] \\ \hat{y}_m(t) = & \sum_{j=1}^{q} \lambda_j(\theta_m(t)) C_{mj} \hat{x}_m(t) \end{cases} \tag{6}$$

$$\text{Slave:} \begin{cases} \dot{\hat{x}}_s(t) = & \sum_{j=1}^{q} \lambda_j(\theta_s(t)) \left[A_{sj} \hat{x}_s(t) + B_{sj} u_s(t) + L_{sj}(y_s(t) - \hat{y}_s(t)) \right] \\ \hat{y}_s(t) = & \sum_{j=1}^{q} \lambda_j(\theta_s(t)) C_{sj} \hat{x}_s(t) \end{cases} \tag{7}$$

where $\hat{x}_i(t) \in \mathbb{R}^n$, $(i = \{m, s\})$ and $\hat{y}_i(t) \in \mathbb{R}^n$ denote the state estimation of $x_i(t) \in \mathbb{R}^n$ and the state observer output, respectively, while $L_{ij} \in \mathbb{R}^{n \times m}$, $i = \{m, s\}$ is the observer gain matrix to be designed.

Let $e_m(t) = x_m(t) - \hat{x}_m(t)$ and $e_s(t) = x_s(t) - \hat{x}_s(t)$ denote the estimation errors.

Lemma 1 (*[30]*). *Let $\bar{x}$ be a differentiable function in $[b, a] \to \mathbb{R}^n$. Then, the following inequality holds such that for a positive definite matrix $Z \in \mathbb{R}^{n \times n}$,*

$$-(b-a) \int_b^a \dot{\bar{x}}^T(s) Z \dot{\bar{x}}(s) \mathrm{d}s \leq \frac{1}{a-b} \Psi_1^T Z \Psi_1 + \frac{3}{a-b} \Psi_2^T Z \Psi_2,$$

where $\Psi_1 = \bar{x}(a) - \bar{x}(b), \Psi_2 = \bar{x}(a) + \bar{x}(b) - \frac{2}{a-b} \int_b^a \bar{x}(s) ds$.

Lemma 2 (*[43]*). *For any vector $\xi \in \mathbb{R}^m$, symmetric matrices $Z_1, Z_2 \in \mathbb{S}_n^+, W_1, W_2 \in \mathbb{R}^{n \times m}$, and any matrices Y_1, Y_2, the following matrix inequality holds:*

$$\frac{1}{\alpha} W_1^T Z_1 W_1 + \frac{1}{1-\alpha} W_2^T Z_2 W_2 \geq \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}^T \begin{bmatrix} Z_1 + (1-\alpha) T_1 & (1-\alpha) Y_1 + \alpha Y_2 \\ * & Z_2 + \alpha T_2 \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}, \tag{8}$$

where $T_1 = Z_1 - Y_2 Z_2^{-1} Y_2^T, T_2 = Z_2 - Y_1^T Z_1^{-1} Y_1$.

Lemma 3 (*[19]*). *Given a matrix $C \in \mathbb{R}^{p \times q}$, where $p < q$ and the matrix has a full row rank ($rank(C) = p$), the single value decomposition (SVD) on C yields:*

$$C = U[S \quad 0] V^T, \tag{9}$$

where the unitary matrices are denoted by $U \in \mathbb{R}^{p \times p}$ and $V \in \mathbb{R}^{q \times q}$, while $S \in \mathbb{R}^{q \times q}$ denotes a diagonal matrix (in which the positive diagonal elements are arranged in a descending order), respectively.

Lemma 4 (*[19]*). *For a given matrix $C \in \mathbb{R}^{p \times q}$ with and full row rank ($rank(C) = p$), and a symmetric matrix denoted as $X \in \mathbb{R}^{q \times q}$, we can have a matrix $\bar{X} \in \mathbb{R}^{p \times p}$ that satisfies $CX = \bar{X}C$ iff X can be expressed as follows:*

$$X = V \begin{bmatrix} X_{11} & 0 \\ 0 & X_{22} \end{bmatrix} V^T, \tag{10}$$

where $X_{11} \in \mathbb{R}^{p \times p}$ and $X_{22} \in \mathbb{R}^{(q-p) \times (q-p)}$ are the unitary matrices of SVD of X.

Remark 2. It can be noted that C in Lemma 4 is an irreversible matrix, which can be represented through the decoupling approach. The significance of this approach is that the equality constraint is avoided; therefore, there is no requirement that matrix C must have full rank. In this respect, studies on the decoupling approach are available in the literature [6,36].

2.3. Integral sliding mode control design

A block diagram of the bilateral teleoperation system with ISMC is shown in Fig. 1. Based on (6) and (7) an observer-based fuzzy ISMC with respect to a T-S fuzzy teleoperation system is developed. To address the communication delays, a delay term $d(t)$ is introduced into the SMC. Consider the sliding surface with the delay as follows,

$$s_i(t) = N_i \hat{x}_i(t) - N_i \int_0^t \sum_{j=1}^{q} \lambda_j(\theta_i(t)) A_{ij} \hat{x}_i(s) \mathrm{d}s - N_i \int_0^t \sum_{j=1}^{q} \lambda_j(\theta_i(t)) B_{ij} K_{ij} (\hat{x}_i(s) - \hat{x}_{\bar{i}}(s - d(s))) \mathrm{d}s, \tag{11}$$

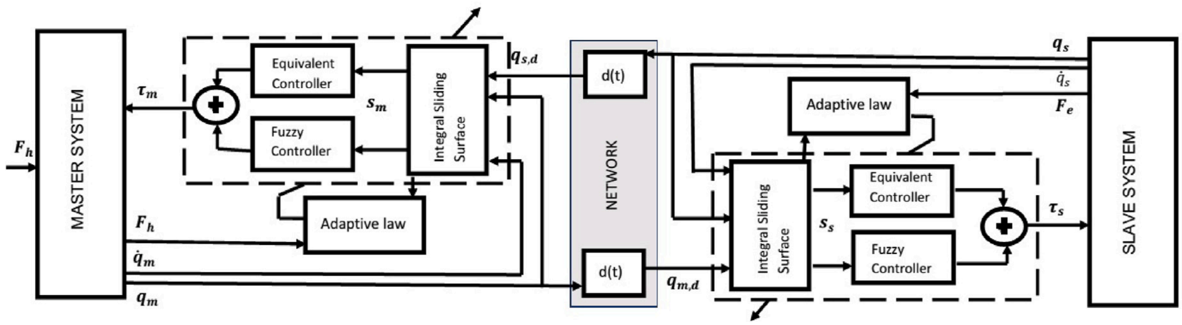

Fig. 1. Representation of bilateral teleoperation system with ISMC.

where $\vec{i}$ is the complementary of i with respect to $i = \{m, s\}$; $N_i \in \mathbb{R}^{m\times n}$ is a parameter matrix satisfying $\det(N_i B_{ij}) \neq 0$; K_{ij} is the unknown control gain matrix to be determined. Combining (6), (7) and (11), the derivative of $s_i(t)$ is of the form,

$$\dot{s}_i(t) = \sum_{j=1}^{q} \lambda_j(\theta_i(t))\Big[N_i B_{ij} u_i(t) + N_i L_{ij} C_{ij} e_i(t) - N_i B_{ij} K_{ij}(\hat{x}_i(t) - \hat{x}_{\bar{i}}(t-d(t)))\Big]. \tag{12}$$

The term $\hat{x}_i(t) - \hat{x}_{\bar{i}}(t-d(t))$ introduced in the sliding surface facilitates the controller formulation. Suppose the time-varying delay $d(t)$ satisfies $0 \le d(t) \le d$, with $-h \le \dot{d}(t) \le h$, where d and h are scalars. In this case, when the state trajectories pertaining to the observer system enters the sliding surface $s_i(t) = 0$, i.e. $\dot{s}_i(t) = 0$, an equivalent control is obtained as

$$u_{i(eq)}(t) = \sum_{j=1}^{q} \lambda_j(\theta_i(t))\Big[-(N_i B_{ij})^{-1} N_i L_{ij} C_{ij} e_i(t) + K_{ij}(\hat{x}_i(t) - \hat{x}_{\bar{i}}(t-d(t)))\Big]. \tag{13}$$

Thus, the fuzzy ISMC can be synthesized as

$$u(t) = u_{i(eq)}(t) - (N_i B_{ij})^{-1}\sigma \mathrm{sign}(s_i(t)), \tag{14}$$

where $\sigma > 0$ is a known scalar. The σsign is used to design the discontinuous function present in the fuzzy ISMC. It is not possible to design the fuzzy ISMC without the usage of σsign. Hence, we have included it in the second term of the controller.

By substituting the developed observer-based fuzzy ISMC (13) into the master–slave observer system (6) and (7), it is obtained that

$$\dot{\hat{x}}_i(t) = \sum_{j=1}^{q} \lambda_j(\theta_i(t))\Big[(A_{ij} + B_{ij}K_{ij})\hat{x}_i(t) - \mathcal{B}_{ij} L_{ij} C_{ij} e_i(t) - B_{ij} K_{ij} \hat{x}_{\bar{i}}(t-d(t))\Big], \tag{15}$$

where $\mathcal{B}_{ij} = B_{ij}(N_i B_{ij})^{-1} N_i - I$. From (4)–(7), the estimation error systems are obtained as follows,

$$\dot{e}_i(t) = \sum_{j=1}^{q} \lambda_j(\theta_i(t))\Big[(A_{ij} - L_{ij}C_{ij})e_i(t) + B_{ij}\digamma_i(t)\Big], \tag{16}$$

where $\digamma_m(t) = F_h(t)$ and $\digamma_s(t) = -F_s(t)$. Let us define $x(t) = [x_m^T(t)\ x_s^T(t)]^T$, $\hat{x}(t) = [\hat{x}_m^T(t)\ \hat{x}_s^T(t)]^T$, $e(t) = [e_m^T(t)\ e_s^T(t)]^T$, $\hat{x}(t-d(t)) = [\hat{x}_m^T(t-d(t))\ \hat{x}_s^T(t-d(t))]^T$.

Therefore, the augmented system for (15) can be obtained as

$$\dot{\hat{x}}(t) = \sum_{j=1}^{q} \lambda_j(\theta(t))\Big[\bar{A}_j \hat{x}(t) - \bar{B}_j e(t) + \bar{D}_j \hat{x}(t-d(t))\Big], \tag{17}$$

where

$$\begin{aligned}
\bar{A}_j &= \begin{bmatrix} A_{mj} + B_{mj}K_{mj} & 0 \\ 0 & A_{sj} + B_{sj}K_{sj} \end{bmatrix}, \\
\bar{B}_j &= \begin{bmatrix} \mathcal{B}_{mj} L_{mj} C_{mj} & 0 \\ 0 & \mathcal{B}_{sj} L_{sj} C_{sj} \end{bmatrix}, \\
\mathcal{B}_{mj} &= (B_{mj}(N_m B_{mj})^{-1} N_m - I), \\
\mathcal{B}_{sj} &= (B_{sj}(N_s B_{sj})^{-1} N_s - I), \\
\bar{D}_j &= \begin{bmatrix} 0 & -B_{mj}K_{mj} \\ -B_{sj}K_{sj} & 0 \end{bmatrix}
\end{aligned}$$

Similarly, we can express (16) as

$$\dot{e}(t) = \sum_{j=1}^{q} \lambda_j(\theta(t))\Big[\bar{A}_j^e e(t) + \bar{E}_j \omega(t)\Big], \tag{18}$$

where $\omega(t) = [F_h(t)\ F_s(t)]^T$,

$$\bar{A}^e_j = \begin{bmatrix} A_{mj} - L_{mj}C_{mj} & 0 \\ 0 & A_{sj} - L_{sj}C_{sj} \end{bmatrix},$$
$$\bar{E}_j = \begin{bmatrix} B_{mj} & 0 \\ 0 & -B_{sj} \end{bmatrix}.$$

Remark 3. It is to be noted that the terms $\hat{x}_m(t)-\hat{x}_s(t-d(t))$ and $\hat{x}_s(t)-\hat{x}_m(t-d(t))$ in the sliding surface (11) provide the position and velocity information with respect to the master–slave robots (i.e. exchange of information between the master and slave systems). Since the problem of time delays is included, the designed observer-based fuzzy ISMC can provide robust synchronization and tracking responses.

3. Stability analysis

We apply the proposed observer-based fuzzy ISMC to analyze the stability condition of the bilateral teleoperation system. During the analysis, some sufficient conditions are derived by employing the delay-product type of LKFs together with Wirtinger-based integral inequality and ERCMI. Besides that, under the zero initial condition, the tracking performance of H_∞ output pertaining to the tracking error $\tilde{e}(t) = y_m(t) - y_s(t)$ of the bilateral teleoperation system is guaranteed in the presence of disturbance attenuation γ, i.e.,

$$\int_0^\infty \tilde{e}^T(s)\tilde{e}(s)\mathrm{d}s \le \gamma^2 \int_0^\infty \omega^T(s)\omega(s)\mathrm{d}s. \tag{19}$$

For convenience, the following notations are used for constructing the delay-product type of LKFs,

$$\xi(t) = \Big[\hat{x}^T(t), \hat{x}^T(t-d(t)), \hat{x}^T(t-d), \frac{1}{d(t)}\int_{t-d(t)}^{t} \hat{x}^T(s)\mathrm{d}s, \frac{1}{d-d(t)}\int_{t-d}^{t-d(t)} \hat{x}^T(s)\mathrm{d}s, \dot{\hat{x}}^T(t), \dot{\hat{x}}^T(t-d(t)),$$
$$\dot{\hat{x}}^T(t-d), e^T(t), \dot{e}^T(t)\Big]^T. \tag{20}$$

As in [31], the positive definite matrices having appropriate dimensions are denoted as R_1, R_2, S_1 and S_2. As such, we can construct the delay-product type of LKFs as:

$$V_r(t) = d(t)\int_{t-d(t)}^{t} \dot{\hat{x}}^T(s)R_1\dot{\hat{x}}(s)\mathrm{d}s + (d-d(t))\int_{t-d}^{t-d(t)} \dot{\hat{x}}^T(s)R_2\dot{\hat{x}}(s)\mathrm{d}s,$$
$$V_s(t) = -\frac{d(t)}{d}\zeta_1^T(t)S_1\zeta_1(t) - \frac{(d-d(t))}{d}\zeta_2^T(t)S_2\zeta_2(t).$$

in which S_1, S_2 are assumed to be

$$S_\mu = \begin{bmatrix} 4R_\mu & 2R_\mu & -6R_\mu \\ * & 4R_\mu & -6R_\mu \\ * & * & 12R_\mu \end{bmatrix}, \quad \mu = 1, 2.$$

and

$$\zeta_1(t) = \Big[\hat{x}^T(t), \hat{x}^T(t-d(t)), \frac{1}{d(t)}\int_{t-d(t)}^{t} \hat{x}^T(s)\mathrm{d}s\Big]^T,$$
$$\zeta_2(t) = \Big[\hat{x}^T(t-d(t)), \hat{x}^T(t-d), \frac{1}{d-d(t)}\int_{t-d}^{t-d(t)} \hat{x}^T(s)\mathrm{d}s\Big]^T \tag{21}$$

Remark 4. Most existing literature studies on the LKF technique considers either a multiple integral functional approach or an augmented LKF methodology that incorporates some available integral inequalities. On the other hand, the information with respect to the time delay and its derivatives can be exploited through the delay-product type terms appearing in the Lyapunov functional. In addition, it consists of both non-integral quadratic terms and integral terms shown in $V_r(t)$ and $V_s(t)$, for which the additional constraints appearing in the non-integral terms could be minimized, resulting in less computational complexity.

Theorem 1. *In the presence of scalars h, d, and $\epsilon_1 > 0, \epsilon_2 > 0$, the closed-loop models (17) and (18) are said to be globally asymptotically stable in the H_∞ sense for a prescribed level γ, subject to the existence of positive definite matrices $P_\mu = diag\{P_{\mu m}, P_{\mu s}\}$, $Q_\mu = diag\{Q_{\mu m}, Q_{\mu s}\}$, $R_\mu = diag\{R_{\mu m}, R_{\mu s}\}$ ($\mu = 1, 2$), $M = diag\{M_m, M_s\} \in \mathbb{R}^{n\times n}$, and any matrix $Y \in \mathbb{R}^{2n\times 2n}$, $G \in \mathbb{R}^{n\times n}$ whereby the LMIs expressed below are satisfied with respect to $j = \{1, \ldots, 9\}$.*

$$M - hR_1 > 0, \quad M - hR_2 > 0, \tag{22}$$

$$\begin{bmatrix} \Xi_j^{\Phi}(0,-h) & E_1^T Y & \epsilon_2 \bar{\bar{E}}_j \\ * & -d\,Z_2(-h) & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{23}$$

$$\begin{bmatrix} \Xi_j^{\Phi}(0,h) & E_1^T Y & \epsilon_2 \bar{\bar{E}}_j \\ * & -d\mathcal{Z}_2(h) & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{24}$$

$$\begin{bmatrix} \Xi_j^{\Phi}(d,-h) & E_2^T Y^T & \epsilon_2 \bar{\bar{E}}_j \\ * & -d\mathcal{Z}_1(-h) & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{25}$$

$$\begin{bmatrix} \Xi_j^{\Phi}(d,h) & E_2^T Y^T & \epsilon_2 \bar{\bar{E}}_j \\ * & -d\mathcal{Z}_1(h) & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{26}$$

where

$$\begin{aligned}
&\Xi_j^{\Phi}(d(t),\dot{d}(t)) = \boldsymbol{\Phi}_1 + \boldsymbol{\Phi}_2 + \boldsymbol{\Phi}_3 + \boldsymbol{\Phi}_4 + \boldsymbol{\Phi}_5, \\
&\boldsymbol{\Phi}_1 = f_1^T P_1 f_6 + f_9^T P_2 f_{10} + (f_1^T + f_9^T)\bar{C}_j (f_1 + f_9), \\
&\bar{C}_j = \begin{bmatrix} C_{mj}^T C_{mj} & -C_{mj}^T C_{sj} \\ -C_{sj}^T C_{mj} & C_{sj}^T C_{sj} \end{bmatrix}, \\
&\boldsymbol{\Phi}_2 = f_1^T (Q_1 + Q_2) f_1 - (1-\dot{d}(t)) f_2^T Q_1 f_2 - f_3^T Q_2 f_3, \\
&\boldsymbol{\Phi}_3 = d(t)\Big[f_6^T R_1 f_6 - (1-\dot{d}(t)) f_7^T R_1 f_7\Big] + (d-d(t))\Big[(1-\dot{d}(t)) f_7^T R_2 f_7 - f_8^T R_2 f_8\Big] - \frac{\dot{d}(t)}{d}(\Pi_1^T S_1 \Pi_1 \\
&\qquad - \Pi_2^T S_2 \Pi_2) - \mathit{Sym}\Big[\frac{1}{d}\Pi_1^T S_1 \Pi_3 + \frac{1}{d}\Pi_2^T S_2 \Pi_4\Big], \\
&\boldsymbol{\Phi}_4 = d f_6^T M f_6 - \frac{1}{d} E^T \mathcal{Z} E, \\
&\boldsymbol{\Phi}_5 = \mathit{Sym}\Big[\epsilon_1 (f_1^T G \bar{A}_j f_1 - f_6^T G f_6) + \epsilon_2 (f_9^T G \bar{A}_j^e f_9 - f_{10}^T G f_{10})\Big] + \epsilon_1 f_1^T G \bar{B}_j f_9 + \epsilon_1 f_1^T G \bar{D}_j f_2 - \epsilon_1 f_1^T G f_6 + \epsilon_1 f_6^T G \bar{B}_j f_9, \\
&f_\kappa = \begin{bmatrix} 0_{n\times(\kappa-1)n} & I_n & 0_{n\times(10-\kappa)n} \end{bmatrix}, \quad \kappa = 1,2,\ldots,10 \\
&\bar{\bar{E}}_j = e_9^T G \bar{E}_j + e_{10}^T G \bar{E}_j, \\
&E = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}, \quad E_\kappa = \begin{bmatrix} f_\kappa - f_{\kappa+1} \\ f_\kappa + f_{\kappa+1} - 2f_{\kappa+3} \end{bmatrix}, \quad \kappa = 1,2 \\
&\mathcal{Z} = \begin{bmatrix} \frac{2d-d(t)}{d}\mathcal{Z}_1(\dot{d}(t)) & Y \\ * & \frac{d+d(t)}{d}\mathcal{Z}_2(\dot{d}(t)) \end{bmatrix}, \\
&\qquad \mathcal{Z}_1(\dot{d}(t)) = \mathit{diag}\left\{(M - \dot{d}(t)R_1), 3(M - \dot{d}(t)R_1)\right\}, \\
&\qquad \mathcal{Z}_2(\dot{d}(t)) = \mathit{diag}\left\{(M + \dot{d}(t)R_2), 3(M + \dot{d}(t)R_2)\right\}, \\
&\Pi_1 = \begin{bmatrix} f_1^T & f_2^T & f_4^T \end{bmatrix}^T, \quad \Pi_2 = \begin{bmatrix} f_2^T & f_3^T & f_5^T \end{bmatrix}^T, \\
&\Pi_3 = \begin{bmatrix} d(t) f_6^T \\ d(t)(1-\dot{d}(t)) f_7^T \\ f_1^T - (1-\dot{d}(t)) f_2^T - \dot{d}(t) f_4^T \end{bmatrix}^T, \\
&\Pi_4 = \begin{bmatrix} (d-d(t))(1-\dot{d}(t)) f_7^T \\ (d-d(t)) f_8^T \\ (1-\dot{d}(t)) f_2^T - f_3^T + \dot{d}(t) f_5^T \end{bmatrix}^T.
\end{aligned} \tag{27}$$

Proof. Consider the LKFs as follows:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t),$$

where

$$\begin{aligned}
V_1(t) &= \hat{x}^T(t) P_1 \hat{x}(t) + e^T(t) P_2 e(t), \\
V_2(t) &= \int_{t-d(t)}^{t} \hat{x}^T(s) Q_1 \hat{x}(s) \mathrm{d}s + \int_{t-d}^{t} \hat{x}^T(s) Q_2 \hat{x}(s) \mathrm{d}s, \\
V_3(t) &= \int_{t-d}^{t}\int_{t+\theta}^{t} \dot{\hat{x}}^T(s) M \dot{\hat{x}}(s) \mathrm{d}s\mathrm{d}\theta, \\
V_4(t) &= d(t)\int_{t-d(t)}^{t} \dot{\hat{x}}^T(s) R_1 \dot{\hat{x}}(s)\mathrm{d}s + (d-d(t))\int_{t-d}^{t-d(t)} \dot{\hat{x}}^T(s) R_2 \dot{\hat{x}}(s)\mathrm{d}s - \frac{d(t)}{d}\zeta_1^T(t) S_1 \zeta_1(t) - \frac{(d-d(t))}{d}\zeta_2^T(t) S_2 \zeta_2(t).
\end{aligned}$$

where $\zeta_1(t), \zeta_2(t)$ and S_1, S_2 are defined in (21).

By taking the time derivative pertaining to $V(t)$,

$$\dot{V}_1(t) = 2\dot{\hat{x}}^T(t)P_1\dot{\hat{x}}(t) + 2e^T(t)P_2\dot{e}(t) = \xi^T(t)\Phi_1\xi(t), \tag{28}$$

$$\dot{V}_2(t) = \hat{x}^T(t)(Q_1+Q_2)\hat{x}(t) - (1-\dot{d}(t))\hat{x}^T(t-d(t))Q_1\hat{x}(t-d(t)) - \hat{x}^T(t-d)Q_2\hat{x}(t-d) = \xi^T(t)\Phi_2\xi(t), \tag{29}$$

$$\dot{V}_3(t) = d\dot{\hat{x}}^T(t)M\dot{\hat{x}}(t) - \int_{t-d}^{t}\dot{\hat{x}}^T(s)M\dot{\hat{x}}(s)\mathrm{d}s = \xi^T(t)(d f_6^T M f_6)\xi(t) - \int_{t-d}^{t}\dot{\hat{x}}^T(s)M\dot{\hat{x}}(s)\mathrm{d}s, \tag{30}$$

$$\begin{aligned}\dot{V}_4(t) =& \dot{d}(t)[\int_{t-d(t)}^{t}\dot{\hat{x}}^T(s)R_1\dot{\hat{x}}(s)\mathrm{d}s - \int_{t-d}^{t-d(t)}\dot{\hat{x}}^T(s)R_2\dot{\hat{x}}(s)\mathrm{d}s] + d(t)\dot{\hat{x}}^T(t)R_1\dot{\hat{x}}(t) - (d-d(t))\dot{\hat{x}}^T(t-d)R_1\dot{\hat{x}}(t-d)\\ &-(1-\dot{d}(t))\Big[\dot{\hat{x}}^T(t-d(t))(d(t)R_1-(d-d(t))R_2)\dot{\hat{x}}(t-d(t))\Big] - \frac{\dot{d}(t)}{d}\zeta_1^T(t)S_1\zeta_1(t) - \frac{2d(t)}{d}\zeta_1^T(t)S_1\dot{\zeta}_1(t)\\ &+\frac{\dot{d}(t)}{d}\zeta_2^T(t)S_2\zeta_2(t) - \frac{2(d-d(t))}{d}\zeta_2^T(t)S_2\dot{\zeta}_2(t)\\ =& \xi^T(t)\Phi_3\xi(t) + \dot{d}(t)\int_{t-d(t)}^{t}\dot{\hat{x}}^T(s)R_1\dot{\hat{x}}(s)\mathrm{d}s - \dot{d}(t)\int_{t-d}^{t-d(t)}\dot{\hat{x}}^T(s)R_2\dot{\hat{x}}(s)\mathrm{d}s,\end{aligned} \tag{31}$$

where $\xi(t)$ is defined in (20), and Φ_1, Φ_2, Φ_3 are defined in (27).

Now, we can express the single integral term appearing in (30) as

$$-\int_{t-d}^{t}\dot{\hat{x}}^T(s)M\dot{\hat{x}}(s)\mathrm{d}s \le -\int_{t-d(t)}^{t}\dot{\hat{x}}^T(s)M\dot{\hat{x}}(s)\mathrm{d}s - \int_{t-d}^{t-d(t)}\dot{\hat{x}}^T(s)M\dot{\hat{x}}(s)\mathrm{d}s \tag{32}$$

Considering the R_1, R_2 and M dependent single integral terms in (31), (32) with $R_1 > 0, R_2 > 0, M > 0$, then one has

$$LMIs \quad (22) \Rightarrow \begin{cases} M - \dot{d}(t)R_1 > 0 \\ M + \dot{d}(t)R_2 > 0. \end{cases}$$

Based on Lemma 1, we can estimate the integral terms as

$$-\int_{t-d(t)}^{t}\dot{\hat{x}}^T(s)(M-\dot{d}(t)R_1)\dot{\hat{x}}(s)\mathrm{d}s \le -\frac{1}{d(t)}\eta_1^T(t)Z_1(\dot{d}(t))\eta_1(t), \tag{33}$$

$$-\int_{t-d}^{t-d(t)}\dot{\hat{x}}^T(s)(M+\dot{d}(t)R_1)\dot{\hat{x}}(s)\mathrm{d}s \le -\frac{1}{d-d(t)}\eta_2^T(t)Z_2(\dot{d}(t))\eta_2(t), \tag{34}$$

where

$$\eta_1(t) = \begin{bmatrix} f_1 - f_2 \\ f_1 + f_2 - 2f_4 \end{bmatrix} = E_1\xi(t),$$

$$\eta_2(t) = \begin{bmatrix} f_2 - f_3 \\ f_2 + f_3 - 2f_5 \end{bmatrix} = E_2\xi(t),$$

and $Z_1(\dot{d}(t))$, $Z_2(\dot{d}(t))$, are defined in (27).

By employing Lemma 2 to the inequalities (33) and (34), it follows that

$$\xi^T(t)[-\frac{1}{d(t)}E_1^T Z_1(\dot{d}(t))E_1 - \frac{1}{d-d(t)}E_2^T Z_2(\dot{d}(t))E_2]\xi(t) \le -\frac{1}{d}\xi^T(t)\begin{bmatrix}E_1\\E_2\end{bmatrix}^T Z \begin{bmatrix}E_1\\E_2\end{bmatrix}\xi(t), \tag{35}$$

where

$$Z = \begin{bmatrix} Z_1(\dot{d}(t)) + \frac{d-d(t)}{d}\varpi_1 & Y \\ * & Z_2(\dot{d}(t)) + \frac{d(t)}{d}\varpi_2 \end{bmatrix},$$

$$\varpi_1 = Z_1(\dot{d}(t)) - YZ_2^{-1}(\dot{d}(t))Y^T,$$

$$\varpi_2 = Z_2(\dot{d}(t)) - Y^T Z_1^{-1}(\dot{d}(t))Y.$$

From the augmented systems (17) and (18), the following equations hold:

$$\sum_{j=1}^{q}\lambda_j(\theta(t))2(\hat{x}(t)G_1 + \dot{\hat{x}}(t)G_1)\Big[\bar{A}_j\hat{x}(t) - \bar{B}_je(t) + \bar{D}_j\hat{x}(t-d(t)) - \dot{\hat{x}}(t)\Big] = 0.$$

$$\sum_{j=1}^{q}\lambda_j(\theta(t))2(e(t)G_2 + \dot{e}(t)G_2)\Big[\bar{A}_j^e e(t) + \bar{E}_j\omega(t) - \dot{e}(t)\Big] = 0. \tag{36}$$

where $G_1 = \epsilon_1 G$, and $G_2 = \epsilon_2 G$, ϵ_1, ϵ_2 being positive scalars.

Then, combining (28)–(36), the derivative of $V(t)$ along the trajectories of (17) and (18) with H_∞ norm (19) is able to satisfy

$$\dot{V}(t)+\tilde{e}^T(t)\tilde{e}(t)-\gamma^2\omega^T(t)\omega(t)\le\sum_{j=1}^{q}\lambda_j(\theta(t))\begin{bmatrix}\xi(t)\\ \omega(t)\end{bmatrix}^T\begin{bmatrix}\Xi_j(d(t),\dot{d}(t)) & \epsilon_2\bar{\bar{E}}_j\\ * & -\gamma^2 I\end{bmatrix}\begin{bmatrix}\xi(t)\\ \omega(t)\end{bmatrix},$$
$$=\sum_{j=1}^{q}\lambda_j(\theta(t))\begin{bmatrix}\xi(t)\\ \omega(t)\end{bmatrix}^T\tilde{\Xi}_j(d(t),\dot{d}(t))\begin{bmatrix}\xi(t)\\ \omega(t)\end{bmatrix}.$$

Note that the linear matrix-valued function, i.e., $\Xi_j(d(t),\dot{d}(t))$, is in a form whereby for all $(d(t),\dot{d}(t))=[0,d]\times[-h,h]$, and with a convex combination technique, we can obtain

$$\begin{cases}\Xi_j(d(t)=0,\dot{d}(t)=-h)<0,\\ \Xi_j(d(t)=0,\dot{d}(t)=h)<0,\\ \Xi_j(d(t)=d,\dot{d}(t)=-h)<0,\\ \Xi_j(d(t)=d,\dot{d}(t)=h)<0.\end{cases}\Rightarrow\Xi_j(d(t),\dot{d}(t))<0 \tag{37}$$

Thus based on Schur complement, one can obtain

$$(23)\Leftrightarrow\tilde{\Xi}_j(0,-h),\quad (24)\Leftrightarrow\tilde{\Xi}_j(0,h),$$
$$(25)\Leftrightarrow\tilde{\Xi}_j(d,-h),\quad (26)\Leftrightarrow\tilde{\Xi}_j(d,h).$$

Therefore, according to Lyapunov stability theory if LMIs (23)–(26) are satisfied, then the augmented systems (17) and (18) are globally asymptotically stable pertaining to H_∞ control in the presence of a disturbance attenuation level γ. The proof is completed.

Remark 5. Owing to nonlinear terms such as $G\bar{A}_j, G\bar{B}_j, G\bar{D}_j, G\bar{A}^e_j$, the LMIs from (22)–(26) in Theorem 1 cannot be implemented. Thus, to calculate the stabilizing gain matrices K_{ij}, L_{ij} and to formulate the corresponding sufficient conditions that are able to guarantee feasibility of the observer and error systems, the SVD is applied in the following theorem.

Theorem 2. *In the presence of scalars h, d, and $\epsilon_1>0,\epsilon_2>0$, the closed-loop systems (17) and (18) are said to be globally asymptotically stable in the H_∞ sense pertaining to a prescribed level γ, subject to the existence of positive definite matrices $\hat{P}_\mu=diag\{\hat{P}_{\mu m},\hat{P}_{\mu s}\}$, $\hat{Q}_\mu=diag\{\hat{Q}_{\mu m},\hat{Q}_{\mu s}\}$, $\hat{R}_\mu=diag\{\hat{R}_{\mu m},\hat{R}_{\mu s}\}$ $(\mu=1,2)$, $\hat{M}=diag\{\hat{M}_m,\hat{M}_s\}, X\in\mathbb{R}^{n\times n}$, any matrix $\hat{Y}\in\mathbb{R}^{2n\times 2n}$ and gain matrices $\hat{K}_{ij},\hat{L}_{ij}$ with appropriate dimensions whereby the LMIs expressed below are satisfied with respect to $i=\{m,s\}, j=\{1,\dots,9\}$.*

$$\hat{M}-h\hat{R}_1>0,\quad \hat{M}-h\hat{R}_2>0, \tag{38}$$

$$\begin{bmatrix}\hat{\Xi}^{\Phi}_j(0,-h) & E_1^T\hat{Y} & \epsilon_2\bar{\bar{E}}_j\\ * & -d\hat{Z}_2(-h) & 0\\ * & * & -\gamma^2 I\end{bmatrix}<0, \tag{39}$$

$$\begin{bmatrix}\hat{\Xi}^{\Phi}_j(0,h) & E_1^T\hat{Y} & \epsilon_2\bar{\bar{E}}_j\\ * & -d\hat{Z}_2(h) & 0\\ * & * & -\gamma^2 I\end{bmatrix}<0, \tag{40}$$

$$\begin{bmatrix}\hat{\Xi}^{\Phi}_j(d,-h) & E_2^T\hat{Y}^T & \epsilon_2\bar{\bar{E}}_j\\ * & -d\hat{Z}_1(-h) & 0\\ * & * & -\gamma^2 I\end{bmatrix}<0, \tag{41}$$

$$\begin{bmatrix}\hat{\Xi}^{\Phi}_j(d,h) & E_2^T\hat{Y}^T & \epsilon_2\bar{\bar{E}}_j\\ * & -d\hat{Z}_1(h) & 0\\ * & * & -\gamma^2 I\end{bmatrix}<0, \tag{42}$$

where

$$\hat{\Xi}^{\Phi}_j(d(t),\dot{d}(t))=\hat{\Phi}_1+\hat{\Phi}_2+\hat{\Phi}_3+\hat{\Phi}_4+\hat{\Phi}_5,$$
$$\hat{\Phi}_1=f_1^T\hat{P}_1f_6+f_9^T\hat{P}_2f_{10}+(f_1^T+f_9^T)\hat{\bar{C}}_j(f_1+f_9),$$
$$\hat{\Phi}_2=f_1^T(\hat{Q}_1+\hat{Q}_2)f_1-(1-\dot{d}(t))f_2^T\hat{Q}_1f_2-f_3^T\hat{Q}_2f_3,$$
$$\hat{\Phi}_3=d(t)\Big[f_6^T\hat{R}_1f_6-(1-\dot{d}(t))f_7^T\hat{R}_1f_7\Big]+(d-d(t))\Big[(1-\dot{d}(t))\times f_7^T\hat{R}_2f_7-f_8^T\hat{R}_2f_8\Big]-\frac{\dot{d}(t)}{d}(\Pi_1^T\hat{S}_1\Pi_1$$
$$-\Pi_2^T\hat{S}_2\Pi_2)-Sym\Big[\frac{1}{d}\Pi_1^T\hat{S}_1\Pi_3+\frac{1}{d}\Pi_2^T\hat{S}_2\Pi_4\Big],$$
$$\hat{\Phi}_4=d f_6^T\hat{M}f_6-\frac{1}{d}E^T\hat{Z}E,$$
$$\hat{\Phi}_5=Sym\Big[\epsilon_1f_1^T\hat{A}_jf_1-\epsilon_1f_6^TXf_6+\epsilon_2f_9^T\hat{A}^e_jf_9-\epsilon_2f_{10}^TXf_{10}\Big]+\epsilon_1f_1^T\hat{B}_jf_9+\epsilon_1f_1^T\hat{D}_jf_2-\epsilon_1f_1^TXf_6+\epsilon_1f_6^T\hat{B}_jf_9,$$

$$f_\kappa = \begin{bmatrix} 0_{n\times(\kappa-1)n} & I_n & 0_{n\times(10-\kappa)n} \end{bmatrix}, \quad \kappa = 1,2,\ldots,10$$
$$\bar{\bar{E}}_j = e_9^T X^{-1}\bar{E}_j + e_{10}^T X^{-1}\bar{E}_j, \quad X^{-1} = diag\{\check{X}^{-1}, \check{X}^{-1}\}$$
$$E = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}, \quad E_\kappa = \begin{bmatrix} f_\kappa - f_{\kappa+1} \\ f_\kappa + f_{\kappa+1} - 2f_{\kappa+3} \end{bmatrix}, \quad \kappa = 1,2$$
$$\hat{\mathcal{Z}} = \begin{bmatrix} \frac{2d-d(t)}{d}\hat{\mathcal{Z}}_1(\dot{d}(t)) & \hat{Y} \\ * & \frac{d+d(t)}{d}\hat{\mathcal{Z}}_2(\dot{d}(t)) \end{bmatrix},$$
$$\hat{\mathcal{Z}}_1(\dot{d}(t)) = diag\left\{(\hat{M} - \dot{d}(t)\hat{R}_1), 3(\hat{M} - \dot{d}(t)\hat{R}_1)\right\},$$
$$\hat{\mathcal{Z}}_2(\dot{d}(t)) = diag\left\{(\hat{M} + \dot{d}(t)\hat{R}_2), 3(\hat{M} + \dot{d}(t)\hat{R}_2)\right\},$$
$$\Pi_1 = \begin{bmatrix} f_1^T & f_2^T & f_4^T \end{bmatrix}^T, \quad \Pi_2 = \begin{bmatrix} f_2^T & f_3^T & f_5^T \end{bmatrix}^T,$$
$$\Pi_3 = \begin{bmatrix} d(t)f_6^T \\ d(t)(1-\dot{d}(t))f_7^T \\ f_1^T - (1-\dot{d}(t))f_2^T - \dot{d}(t)f_4^T \end{bmatrix}^T, \Pi_4 = \begin{bmatrix} (d-d(t))(1-\dot{d}(t))f_7^T \\ (d-d(t))f_8^T \\ (1-\dot{d}(t))f_2^T - f_3^T + \dot{d}(t)f_5^T \end{bmatrix}^T,$$
$$\hat{\hat{A}}_j = \begin{bmatrix} A_{mj}\check{X} + B_{mj}\hat{K}_{mj} & 0 \\ 0 & A_{sj}\check{X} + B_{sj}\hat{K}_{sj} \end{bmatrix},$$
$$\hat{\hat{B}}_j = \begin{bmatrix} \mathcal{B}_{mj}\hat{L}_{mj}C_{mj} & 0 \\ 0 & \mathcal{B}_{sj}\hat{L}_{sj}C_{sj} \end{bmatrix},$$
$$\hat{\hat{D}}_j = \begin{bmatrix} 0 & -B_{mj}\hat{K}_{mj} \\ -B_{sj}\hat{K}_{sj} & 0 \end{bmatrix},$$
$$\hat{\hat{A}}_j^e = \begin{bmatrix} A_{mj}\check{X} - \hat{L}_{mj}C_{mj} & 0 \\ 0 & A_{sj}\check{X} - \hat{L}_{sj}C_{sj} \end{bmatrix}.$$

Furthermore, the SVD with respect to full row rank matrix C is described as $C = U[S \quad 0]V^T$, where the unitary matrices and positive definite matrix are denoted as UU^T, VV^T and S, respectively.

Then we can obtain the controller gain matrices and the associated observer, i.e.,

$$K_{ij} = \hat{K}_{ij}\check{X}^{-1}, \quad L_{ij} = \hat{L}_{ij}US\check{X}_{11}^{-1}S^{-1}U^{-1}, \quad i = \{m,s\}. \tag{43}$$

Proof. Define $G = \mathrm{diag}\{\bar{G},\bar{G}\}$, and let $\check{X} = \bar{G}^{-1}$ implies $\mathrm{diag}\{\check{X},\check{X}\} = X$. Then, the following matrices can be defined: $XP_1X = \hat{P}_1, XP_2X = \hat{P}_2, XQ_1X = \hat{Q}_1, XQ_2X = \hat{q}_2, XR_1X = \hat{R}_1, XR_2X = \hat{R}_2, XMX = \hat{M}, X\bar{C}_jX = \hat{\bar{C}}_j$. Based on Lemmas 3 and 4, for a given matrix $\check{X} = V\begin{bmatrix}\check{X}_{11} & 0 \\ 0 & \check{X}_{12}\end{bmatrix}V^T$ there exists $\bar{\bar{X}} = US\check{X}_{11}S^{-1}U^{-1}$ such that $C\check{X} = \bar{\bar{X}}C$, where $C_{ij} = C$ with $\mathrm{rank}(C) = p$ and $\bar{\bar{X}}^{-1} = US\check{X}_{11}^{-1}S^{-1}U^{-1}$. Then, pre- and post-multiplying $\mathrm{diag}\{X,\ldots,X\}$ to the function $\Xi_j(d(t),\dot{d}(t))$ in (37) of Theorem 1, and based on a convex combination technique and Schur complement, the LMIs (38)–(42) can be obtained. The proof is completed.

The reachability of the sliding surface $s_i(t) = 0, i = \{m,s\}$ is confirmed when, given a finite time, the trajectories pertaining to observer (15) can be driven by the synthesized ISMC (14) onto the specified surface $s_i(t) = 0$.

Theorem 3. *Consider the observer system (15) along with the sliding surface function given in (11). Then, with the synthesized control law (14), the reachability of the sliding mode surface $s_i(t) = 0, i = \{m,s\}$ can be guaranteed within a specific finite time interval.*

Proof. Define $\mathcal{V}(t) = \sum_{i=m,s}\frac{1}{2}s_i^T(t)s_i(t)$ as the Lyapunov function. As such, the time derivative pertaining to $\mathcal{V}(t)$ along the trajectories of sliding mode dynamics (12) is given as,

$$\dot{\mathcal{V}}(t) = \sum_{i=m,s} s_i^T(t)\dot{s}_i(t)$$
$$= \sum_{i=m,s} s_i^T(t)\sum_{j=1}^{q}\lambda_j(\theta_i(t))\Big[N_iB_{ij}u_i(t) + N_iL_{ij}C_{ij}\times e_i(t) - N_iB_{ij}K_{ij}(\hat{x}_i(t) - K_{ij}\hat{x}_{\bar{i}}(t-d(t)))\Big].$$

Applying (14) into $\dot{\mathcal{V}}(t)$, it yields

$$\dot{\mathcal{V}}(t) = \sum_{i=m,s} s_i^T(t)\sum_{j=1}^{q}\lambda_j(\theta_i(t))\Bigg[N_iB_{ij}\Big\{-(N_iB_{ij})^{-1}\times N_iL_{ij}C_{ij}e_i(t) + K_{ij}(\hat{x}_i(t) - \hat{x}_{\bar{i}}(t-d(t)))$$
$$-\,(N_iB_{ij})^{-1}\sigma sign(s_i(t))\Big\} + N_iL_{ij}C_{ij}e_i(t) - N_iB_{ij}K_{ij}(\hat{x}_i(t) - \hat{x}_{\bar{i}}(t-d(t)))\Bigg],$$
$$\leq \sum_{i=m,s} -\sigma\|s_i(t)\|.$$

Apparently, it can be concluded that $\dot{\mathcal{V}}(t) < 0$ iff $s_i(t) \neq 0$, and the fuzzy ISMC law drives the observer system onto the sliding surface $s_i(t) = 0$.

4. Simulation studies and results

To evaluate and demonstrate the usefulness of the formulated controller, we conduct simulations using a teleoperation system comprising robots with 2DOFs in the form of single master and single slave. Assuming that both master–slave manipulators are subject to identical link masses as well as lengths, their dynamics can be described with reference to [12]: $i = \{m, s\}$

$$M_i(q_i) = \begin{bmatrix} (m_1 + m_2)l_1^2 & m_2 l_1 l_2(\mathfrak{s}_1\mathfrak{s}_2 + \mathfrak{c}_1\mathfrak{c}_2) \\ m_2 l_1 l_2(\mathfrak{s}_1\mathfrak{s}_2 + \mathfrak{c}_1\mathfrak{c}_2) & m_2 l_2^2 \end{bmatrix},$$

$$C_i(q_i, \dot{q}_i) = m_2 l_1 l_2(\mathfrak{c}_1\mathfrak{s}_2 - \mathfrak{s}_1\mathfrak{c}_2) \begin{bmatrix} 0 & -\dot{q}_{i2} \\ -\dot{q}_{i1} & 0 \end{bmatrix},$$

$$G_i(q_i) = \begin{bmatrix} -(m_1 + m_2)l_1 g\mathfrak{s}_1 \\ -m_2 l_2 g\mathfrak{s}_2 \end{bmatrix},$$

where $\mathfrak{s}_1 = \sin(q_{i1}), \mathfrak{s}_2 = \sin(q_{i2}), \mathfrak{c}_1 = \cos(q_{i1}), \mathfrak{c}_2 = \cos(q_{i2})$. The values of $f_1(x_i)$, $f_2(x_i)$, $g_{11}(x_i), g_{12}(x_i)$, $g_{21}(x_i), g_{22}(x_i)$ in (3) are given as in [33].

Let the link masses and lengths be $m_1 = m_2 = 1$ kg, $l_1 = l_2 = 1$ m, $g = 9.8$ m/s^2. Following [33,37], the T-S model with triangular membership functions equipped with nine fuzzy rules is used. The angular positions $q_{i1}(t)$ and $q_{i2}(t)$ are constrained within $[-\frac{\pi}{2}, \frac{\pi}{2}]$. For $x_{i1} = q_{i1}, x_{i2} = \dot{q}_{i1}, x_{i3} = q_{i2}, x_{i4} = \dot{q}_{i2}$, the nine fuzzy rules for the T-S fuzzy system (3) are:

Rl: If x_{i1} is $-\frac{\pi}{2}$ and x_{i3} is $-\frac{\pi}{2}$

$$\text{then } \dot{x}_i(t) = A_{i1}x_i(t) + B_{i1}\tau_i(t)$$

R2: If x_{i1} is $-\frac{\pi}{2}$ and x_{i3} is 0

$$\text{then } \dot{x}_i(t) = A_{i2}x_i(t) + B_{i2}\tau_i(t)$$

R3: If x_{i1} is $-\frac{\pi}{2}$ and x_{i3} is $\frac{\pi}{2}$

$$\text{then } \dot{x}_i(t) = A_{i3}x_i(t) + B_{i3}\tau_i(t)$$

R4: If x_{i1} is 0 and x_{i3} is $-\frac{\pi}{2}$

$$\text{then } \dot{x}_i(t) = A_{i4}x_i(t) + B_{i4}\tau_i(t)$$

R5: If x_{i1} is 0 and x_{i3} is 0

$$\text{then } \dot{x}_i(t) = A_{i5}x_i(t) + B_{i5}\tau_i(t)$$

R6: If x_{i1} is 0 and x_{i3} is $\frac{\pi}{2}$

$$\text{then } \dot{x}_i(t) = A_{i6}x_i(t) + B_{i6}\tau_i(t)$$

R7: If x_{i1} is $\frac{\pi}{2}$ and x_{i3} is $-\frac{\pi}{2}$

$$\text{then } \dot{x}_i(t) = A_{i7}x_i(t) + B_{i7}\tau_i(t)$$

R8: If x_{i1} is $\frac{\pi}{2}$ and x_{i3} is 0

$$\text{then } \dot{x}_i(t) = A_{i8}x_i(t) + B_{i8}\tau_i(t)$$

R9: If x_{i1} is $\frac{\pi}{2}$ and x_{i3} is $\frac{\pi}{2}$

$$\text{then } \dot{x}_i(t) = A_{i9}x_i(t) + B_{i9}\tau_i(t)$$

The system output is $y_i(t) = C_{ij}x_i(t)$, $j = 1, \ldots, 9$.

Since the link masses and lengths pertaining to the master–slave manipulators are identical, matrices A_{ij}, B_{ij} and C_{ij} for the nine fuzzy rules can be formed as in [33]. To verify the stability conditions of Theorem 2, choose $d(t) = 0.3\sin^2(0.01t)$. With respect to the time-varying delay, let its upper bound be $d = 1.53$ and its derivative be $h = 0.6$. Then, matrix N_i can be chosen appropriately to satisfy $\det(N_i B_{ij}) \neq 0$. The LMIs (38)–(42) are verified with the use of MATLAB toolbox and the control gain matrices K_{ij} are obtained as,

$$K_{i1} = \begin{bmatrix} 2.9276 & -0.0201 & -0.0522 & -0.0179 \\ 1.9489 & -0.0134 & -0.0341 & -0.0121 \end{bmatrix} \times 10^2,$$

$$K_{i2} = \begin{bmatrix} 2.7416 & 0.5465 & 0.1239 & -3.4613 \\ -4.5770 & 0.3152 & -0.4927 & -1.6540 \end{bmatrix} \times 10^2,$$

$$K_{i3} = \begin{bmatrix} -0.5307 & 0.0459 & 0.2890 & -0.3472 \\ -0.6934 & 0.0635 & 0.8361 & -0.4887 \end{bmatrix} \times 10^2,$$

$$K_{i4} = \begin{bmatrix} -0.2829 & 0.2989 & -9.8695 & 0.9158 \\ -6.2394 & -0.2190 & -0.5839 & 3.8175 \end{bmatrix} \times 10^2,$$

$$K_{i5} = \begin{bmatrix} 2.3863 & -0.0167 & -0.0402 & -0.0129 \\ 1.6838 & -0.0117 & -0.0270 & -0.0093 \end{bmatrix} \times 10^2,$$

$$K_{i6} = \begin{bmatrix} -2.4012 & -0.2296 & 8.0544 & 2.6420 \\ -4.3797 & 0.1331 & 3.0215 & -0.0558 \end{bmatrix} \times 10^2,$$

$$K_{i7} = \begin{bmatrix} 0.6654 & 0.0517 & -0.6123 & -0.2851 \\ -2.6874 & 0.0224 & 0.7263 & -0.0230 \end{bmatrix} \times 10^2,$$

$$K_{i8} = \begin{bmatrix} 1.7872 & 0.9337 & 10.4899 & -7.7219 \\ -4.7726 & 0.1459 & 5.7223 & -0.2118 \end{bmatrix} \times 10^2,$$

$$K_{i9} = \begin{bmatrix} 2.3617 & -0.0166 & -0.0433 & -0.0122 \\ 1.6673 & -0.0117 & -0.0301 & -0.0086 \end{bmatrix} \times 10^2.$$

Similarly the observer gain matrices L_{ij} are obtained as,

$$L_{i1} = \begin{bmatrix} -1.9002 & -1.1015 \\ 1.6003 & -0.1973 \\ 4.0017 & 4.0397 \\ -2.0015 & -5.0418 \end{bmatrix}, L_{i2} = \begin{bmatrix} -1.0287 & -1.6403 \\ 1.2180 & -0.3356 \\ -4.6403 & 4.2180 \\ -2.0715 & -4.5210 \end{bmatrix},$$

$$L_{i3} = \begin{bmatrix} -13.8012 & -6.7092 \\ 7.0006 & 3.0042 \\ -12.0009 & -3.0048 \\ 4.0003 & 1.0016 \end{bmatrix}, L_{i4} = \begin{bmatrix} 14.2034 & 1.9509 \\ -8.0021 & -2.0355 \\ 18.0057 & 9.1128 \\ -6.0019 & -3.0376 \end{bmatrix}$$

$$L_{i5} = \begin{bmatrix} -1.0001 & -1.0012 \\ 5.0006 & 5.0062 \\ -8.0006 & -2.0032 \\ -2.0003 & -3.0036 \end{bmatrix}, L_{i6} = \begin{bmatrix} 1.0003 & 1.2088 \\ -5.218 & 5.3640 \\ 12.0038 & 6.0752 \\ -2.0015 & -5.0418 \end{bmatrix},$$

$$L_{i7} = \begin{bmatrix} -6.6005 - 2.4035 \\ -5.0003 - 0.0006 \\ -9.0009 - 6.0078 \\ - - 2.0003 - 3.0036 \end{bmatrix}, L_{i8} = \begin{bmatrix} 8.4024 & 3.0439 \\ 10.0036 & 7.0773 \\ 6.0006 & -3.0063 \\ -2.0015 & -5.0418 \end{bmatrix},$$

$$L_{i9} = \begin{bmatrix} -1.0001 & -1.0012 \\ -4.0003 & -1.0016 \\ 12.0009 & 3.0048 \\ -2.0003 & -3.0036 \end{bmatrix}.$$

We can obtain the level of disturbance attenuation at $\gamma = 2.303$. The torque applied to the master device is $F_h(t) = J_m^T[0 \quad 1]^T f_m$, where the force of human operator is f_m while the master Jacobian matrix is J_m. In addition, the environmental contact force exerted on the slave by a stiff wall at $y = 0.5$ m is $F_s(t) = J_s^T[0 \quad 1]^T 10000(y - 0.5)$ N m, where J_s is the slave Jacobian matrix. The position and velocity tracking responses with respect to the master–slave robots are depicted in Figs. 2–5, respectively. On the other hand, the responses of the proposed sliding surfaces and the corresponding controller responses are respectively shown in Figs. 6 and 7. Finally, Fig. 8 depicts the estimated error responses of the state trajectories (solid line) and the respective observer trajectories (dotted line) converging to zero. From Figs. 2 to 8, we can infer that by using the proposed observer based fuzzy ISMC, both the master and slave robot are able to attain synchronization in a short span, as compared with those from other existing methods. It should also be noted that good results in terms of velocity and position tracking responses have been obtained. The above figures demonstrate that all slaves are capable of following the master's position and velocity signals. Transparency is therefore achieved. Therefore, the proposed control strategy successfully enhanced the master–slave tracking and synchronization performances.

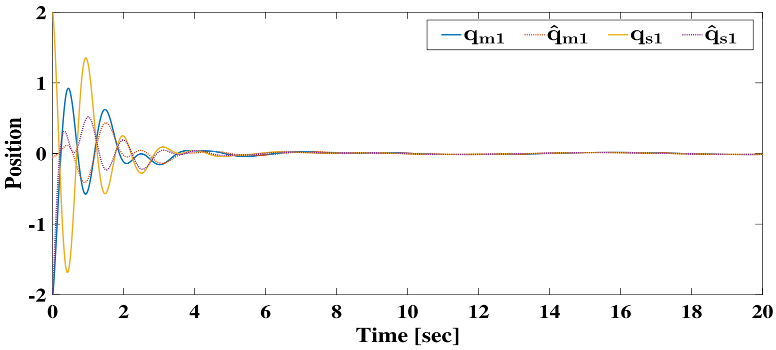

Fig. 2. Position tracking responses of the master and slave manipulators $(q_{i1}, \hat{q}_{i1})$.

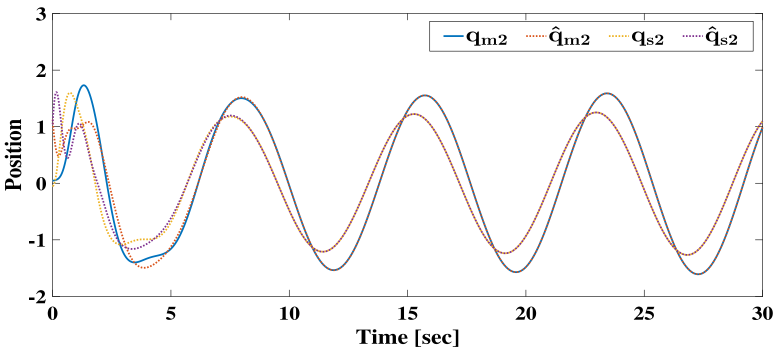

Fig. 3. Position tracking responses of the master–slave manipulators $(q_{i2}, \hat{q}_{i2})$.

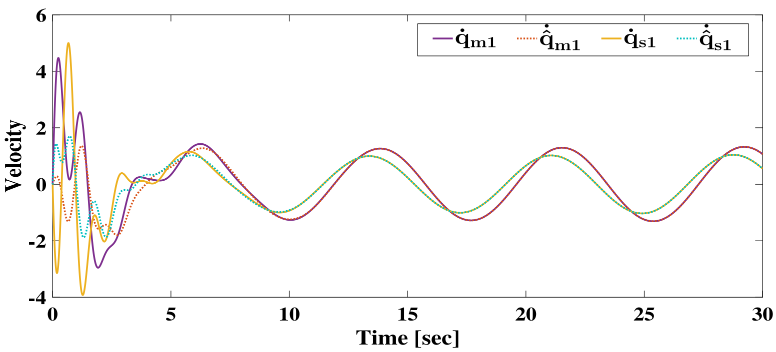

Fig. 4. Velocity tracking responses of the master and slave manipulators $(\dot{q}_{i1}, \dot{\hat{q}}_{i1})$.

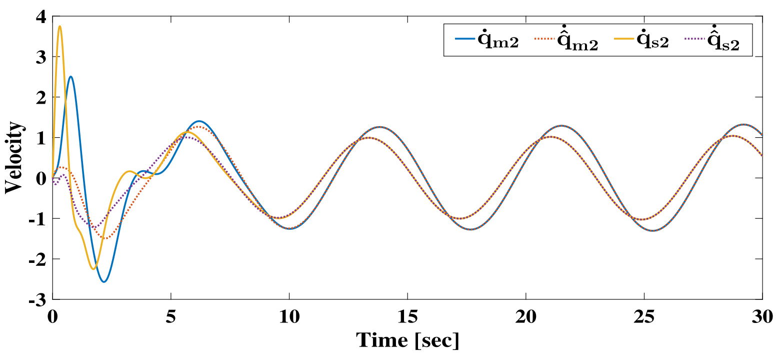

Fig. 5. Velocity tracking responses of the master and slave manipulators $(\dot{q}_{i2}, \dot{\hat{q}}_{i2})$.

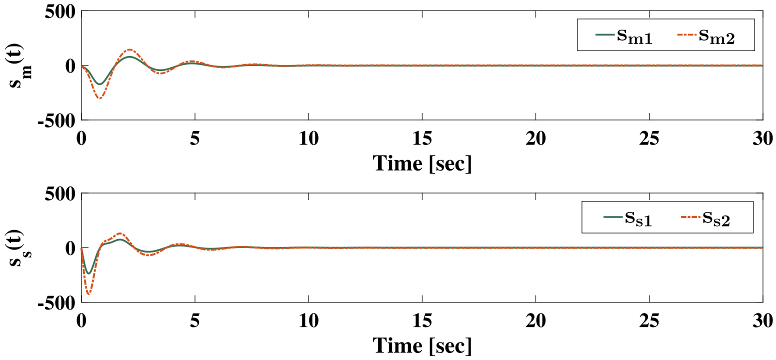

Fig. 6. Master–Slave sliding surface responses ($s_i(t)$).

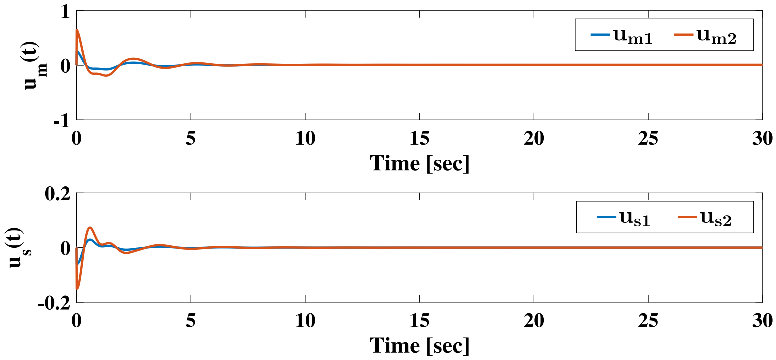

Fig. 7. Master–Slave Control responses ($u_i(t)$).

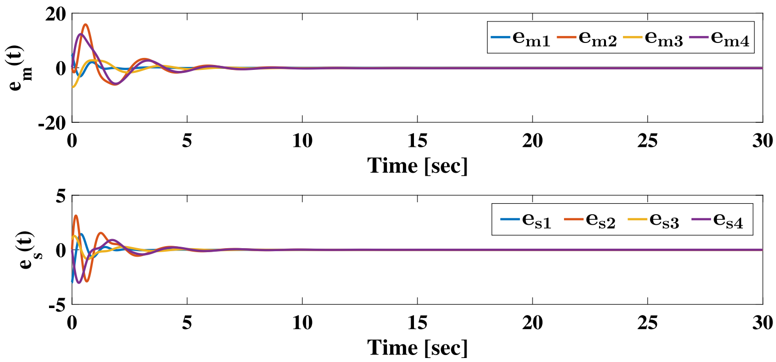

Fig. 8. The estimation error responses ($e_i(t)$).

5. Conclusion

The tracking performance and stability analysis of a nonlinear bilateral telerobotic system with time-varying delays has been studied. The T–S fuzzy master–slave dynamics have been provided with respect to the state observers for which an observer-based fuzzy ISMC has been developed. We have also derived the delay-dependent stability conditions with the Wirtinger based integral inequality and ERCMI based on the use of LKF derivatives. Moreover, the obtained stability conditions expressed through LMIs have been analyzed comprehensively using the SVD method; therefore the system's feasibility is guaranteed. Finally, the provided numerical simulations illustrate efficacy of the proposed method. While the simulation results are promising, it is necessary to validate efficacy of our proposed method in real-world environments. Besides that, from theoretical results presented in this study, it can be seen that the matrix dimension impacts the computational complexity of the algorithm. If a higher matrix dimension is considered, a higher computation cost and time is required, which can potentially affect real-time application of our proposed method in real environments. In our future work, we will conduct actual hardware implementation and experimentation to validate the proposed method in real-world environments.

CRediT authorship contribution statement

K. Janani: Writing – original draft, Reading. **R. Baranitha:** Conceptualization, Simulation, Writing – original draft. **Chee Peng Lim:** Writing – review & editing. **R. Rakkiyappan:** Writing – review & editing, Supervision.

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